FASTOP: A Flutter and Strength Optimization Program for Lifting-Surface Structures

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A new computer program entitled FASTOP is described, and results obtained from its application to the structural sizing of three lifting-surface structures for combined strength and flutter-speed requirements are presented. Two detailed finite-element models of metallic structures (having between 600 and 900 elements) and a preliminary-design representation of an advanced-composite wing are considered. Near-minimum-weight designs are achieved in only six combined strength and flutter resizing cycles for the two metallic structures, one of which includes mass-balance design variables. For the composite wing, FASTOP is used to resize the individual ply thicknesses of a strength-based design for increased flutter speed; the same excellent convergence characteristics are demonstrated.

Introduction

GRUMMAN recently has completed the development of a computer program system entitled FASTOP[1] (Flutter and Strength Optimization Program) which is capable of performing both integrated interdisciplinary analyses and efficient (near-minimum-weight) sizing of large- or small-scale structures models in the presence of both strength and flutter-speed constraints. The system's strength analysis and resizing module is based on a computer program developed several years ago for the Air Force Flight Dynamics Laboratory. That program, known as ASOP[2] (Automated Structural Optimization Program), automatically resizes the gages of a finite-element structures model to achieve a fully stressed (near-minimum-weight) design (FSD), that is, a design in which each element is either subjected to its maximum allowable stress under at least one loading condition, or is at a prespecified minimum permitted gage. The major objective in the creation of FASTOP was to develop an automated procedure for minimum-weight resizing to satisfy a single flutter-speed constraint[3] and to integrate this procedure with ASOP. This need arose because of inefficiencies in existing flutter-prevention procedures, wherein a flutter analyst relies largely on judgment in pursuing a flutter "fix." That approach can require many trial-and-error studies which are costly in time, particularly when large-scale finite-element models are involved.

This paper briefly describes the organization of the FASTOP system and summarizes the capabilities of the various analysis and redesign modules. The system's separate strength and flutter resizing algorithms and the interactive approach to the combined strength/flutter optimization problem then are considered in some detail. The main body of the paper presents the results of studies carried out on three significantly different lifting-surface designs which were chosen to demonstrate the broad redesign capability of FASTOP.

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Overview of the FASTOP System

A functional flow diagram of FASTOP is presented in Fig. 1. The package is comprised of two major programs, which are executed sequentially, with each program designed to perform successive analysis and resizing functions in a single computer submission. The Strength Optimization Program (SOP) focuses on basic aspects of static structural analysis and minimum-weight design for strength requirements. It provides for automated calculation of rigid-surface applied loads, performance of conventional strength and flexibility (or stiffness) analysis, and automated resizing of a structural

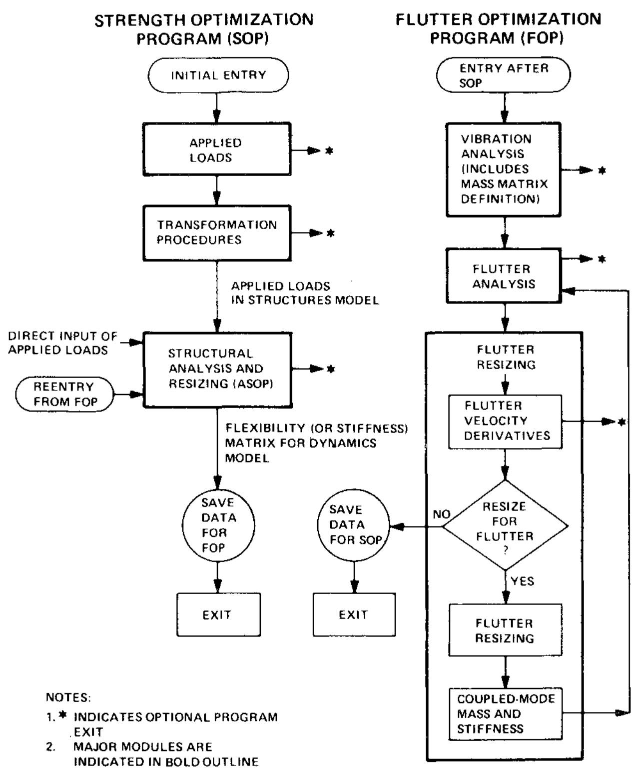

Fig. 1 Functional flow diagram for the FASTOP system.

Table 1 Analysis and redesign capabilities of FASTOP

Applied Loads	
• Aerodynamic	
– maximum number of flight conditions (subsonic and/or supersonic)	8
• Inertial	
– maximum number of flight conditions	8
Structural Analysis and Resizing	
• Primarily for metallic structures but with limited composites capability	
– maximum number of finite elements	3000
– maximum number of structures model degrees of freedom	6000
– maximum number of applied loading conditions	8
Vibration Analysis	
• Applicable to cantilever or free-free structures	
– maximum number of dynamics model degrees of freedom	200
Flutter Analysis	
• Assumed-pressure function and doublet-lattice routines for subsonic flow	M = 0 ⟶ 0.9
• Mach-box routine for supersonic flow	M = 1.2 ⟶ 3.0
– maximum number of modes	20
Flutter Resizing	
• Applicable to metallic and composite structures	
– maximum number of flutter design variables:	
o structural elements	2000
o mass-balance elements	20

idealization to achieve a fully stressed design. It also prepares data required for direct input to the second major program. The Flutter Optimization Program (FOP) addresses dynamic analysis requirements and provides the redesign capability for achieving a desired value of flutter speed with minimum cost in weight. Using output data from the first program, it proceeds to establish mass matrix input for vibration mode analysis, compute normal mode shapes and frequencies, determine the surface's critical flutter speed, and, if desired, perform resizing to increase the flutter speed. Finally, the second program saves data required for re-entering SOP. The capabilities of the various analysis and redesign modules of both programs are summarized in Table 1.

Strength and Flutter Resizing

The separate resizing algorithms used to satisfy strength and flutter-speed requirements are summarized first. The blending of these two resizing techniques into an interactive strength/flutter resizing approach is discussed subsequently.

Strength Resizing Algorithm

Strength resizing in FASTOP is based on the concept of a fully stressed design; that is, the structure is sized so that each element either is subjected to its maximum allowable stress under at least one of the applied loading conditions or is limited by a prespecified minimum-gage constraint. Such a design is achieved by performing standard repetitive structural analyses for internal finite-element loads and effective stresses, followed by element resizing. In each resizing step, the internal element loads are assumed to remain constant; this suggests that the element's gage be multiplied by the maximum ratio of actual-to-allowable stress for all loading conditions. Of course, this is subject to the limitaiton that no element is ever permitted to be reduced in size below its minimum specified value. For statically determinate structures, the assumption of constant internal loads is valid, and the fully stressed design is obtained in a single resizing step. However, for indeterminate structures, the process must be repeated several times before convergence is achieved.

Flutter Resizing Algorithm

The procedure employed to resize the structure to meet a minimum flutter-speed requirement is based on the criterion that, for minimum weight, the derivatives of the flutter speed with respect to element weight must be equal for all elements that have been resized to meet the flutter-speed requirement. This criterion, and the development of a resizing equation that achieves the uniform-derivative state for both structural and mass-balance design variables, is discussed in detail in Ref. 3. Equations for the required flutter-velocity derivatives, as they are programmed in FASTOP, are included in that reference, and appear in their original form in Ref. 4.

Letting $m_{i_{\text{old}}}$ and $m_{i_{\text{new}}}$ denote the respective weights of the ith element, before and after resizing, the flutter resizing equation is

$$m_{i_{\text{new}}} = m_{i_{\text{old}}} \sqrt{\frac{(\partial V_f/\partial m_i)_{\text{old}}}{(\partial V_f/\partial m_i)_{\text{target}}}} \tag{1}$$

where $(\partial V_f/\partial m_i)_{\text{old}}$ is the derivative of the flutter velocity, V_f, with respect to the weight of the ith element, prior to resizing; the "target" derivative in the denominator of the equation is the desired uniform value sought for all of the resized elements.

In applying this resizing approach an iterative procedure is employed to determine the value of the target derivative used

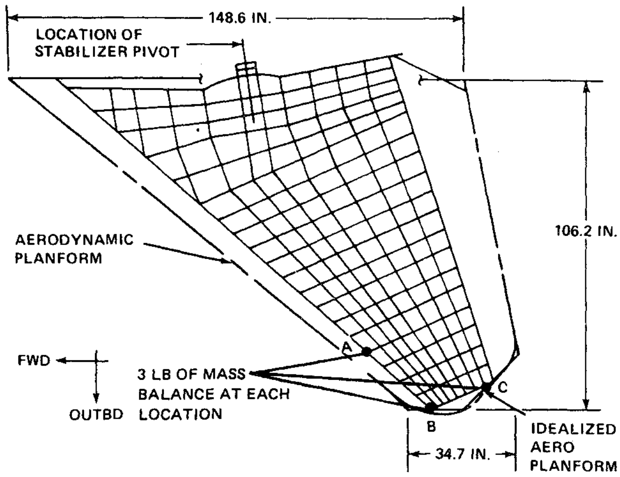

Fig. 2 All-movable stabilizer structures model.

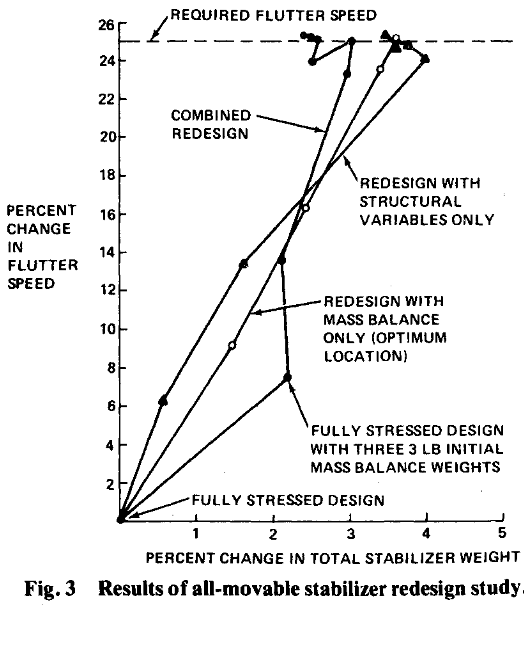

Fig. 3 Results of all-movable stabilizer redesign study.

in Eq. (1). Starting with an assumed value for the target derivative, Eq. (1) is used to obtain a tentative new design. The flutter-speed increment associated with this "trial" redesign then is approximated by the linear relationship

$$\delta V_f = \sum_i \left(\frac{\partial V_f}{\partial m_i} \right) (m_{i_{\text{new}}} - m_{i_{\text{old}}}) \tag{2}$$

If this predicted increment is within a specified tolerance of the desired flutter-speed increment, the trial redesign is accepted. Otherwise, the target derivative is adjusted automatically and the process is repeated.

As a matter of practical interest, some comments are in order regarding the use of Eq. (1). Many element derivatives will be very small, or even negative, and in such cases it would appear desirable to reduce the element's size to its value dictated by strength or minimum-gage requirements. However, in some cases, it was found that the stability of the resizing procedure was improved if the reduction in any element's size, in a redesign step, was limited to a specified percentage of its previous value.

Interactive Resizing in FASTOP

The two-program approach described earlier indicates that strength resizing and flutter resizing are carried out separately. However, the process is actually interactive, because the two redesign modules exchange data and impose restrictions on each other. This interactive redesign of a lifting-surface structure proceeds as follows.

Starting with the use of the Structural Optimization Program (SOP), the structure is sized to satisfy its strength requirements with a fully stressed design. Two categories of elements exist after this first step; specifically, each element either is fully stressed (i.e., "strength-critical") or is at its prespecified minimum gage (as dictated, for example, by manufacturing considerations). The next step uses the Flutter Optimization Program (FOP) to resize structural elements and/or mass-balance design variables to increase the surface's critical flutter speed. None of the structural elements is permitted to be reduced in size in this initial FOP step since, upon entering the step, all of these elements were already either strength-critical or at prespecified minimum gage. Those structural elements that are increased in size in this step, plus any mass-balance variables present in the design, constitute a third category of elements, namely, "flutter-critical" elements, i.e., elements whose gages are dictated by flutter-speed requirements.

The resizing of some structural elements during the first FOP step may cause a significant redistribution of internal loads within the structure, thereby modifying gage requirements for strength considerations. Accordingly, in the next step, SOP is re-entered for the purpose of a "strength update." This is the first attempt to achieve a minimum-weight design that accounts for strength/flutter interaction. In this second SOP step, the flutter-critical elements (and, of course, the minimum-gage elements) are not permitted to be resized downward, but the strength-critical elements are free to be sized upward or downward. After resizing, the various elements are reclassified into strength-critical, flutter-critical, and minimum-gage categories. Elements may, of course, change categories. For example, an element would shift from the flutter-critical to the strength-critical set if it had been resized upward to satisfy the modified strength requirements. Likewise, an element which was previously strength-critical might be resized downward to the point where it enters the minimum-gage category.

At this stage, FOP is entered for the second time and the interactive strength/flutter redesign process continues. As in the first FOP step, elements in the strength-critical and minimum-gage categories cannot be sized downward. On the other hand, there now exists a set of elements, namely, the flutter-critical elements, which are free to be resized in either direction; if such an element is sized downward, however, its gage is not permitted to fall below the values required by the last strength analysis or by minimum-gage considerations. After resizing, the elements again are reclassified into the three basic categories.

Subsequent interactive application of the two programs proceeds in a manner similar to the second SOP and FOP steps until the process is sufficiently converged. The final design will consist of a set of flutter-critical elements which have nearly uniform flutter-velocity derivatives, a set of strength-critical elements which are fully stressed, and a set of elements which are at prespecified minimum gages.

Examples of the Use of FASTOP

Calculations to demonstrate the redesign capability of FASTOP were performed using detailed finite-element models of two metallic lifting-surface structures, namely, an all-movable horizontal stabilizer and a wing with a pylon-mounted store. A third study was conducted using a simpler preliminary-design representation of a wing with advanced-composite cover skins and aluminum substructure. The characteristics of these models and the results obtained from the application of FASTOP are described in the following.

For the first two demonstration cases, FASTOP was used initially to 1) compute aerodynamic design loads for specified flight conditions, 2) distribute the design loads to the structures model, and 3) resize the initial gages of the structures model to obtain a fully stressed design (FSD). The program subsequently was used to perform interactive strength and flutter resizing beyond the initial strength-based

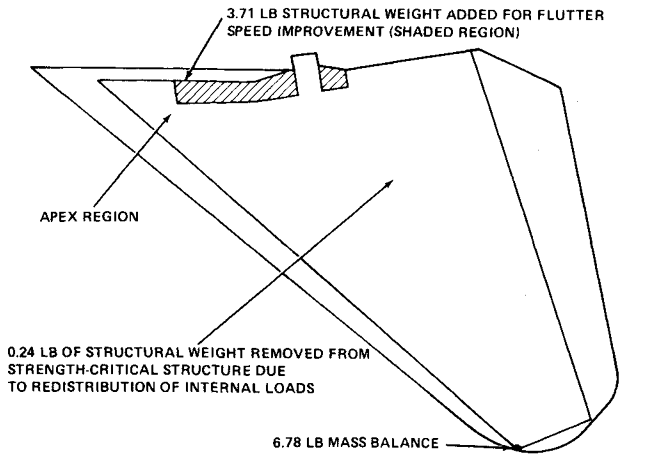

Fig. 4 Design changes in all-movable stabilizer with combined structural and mass-balance design variables.

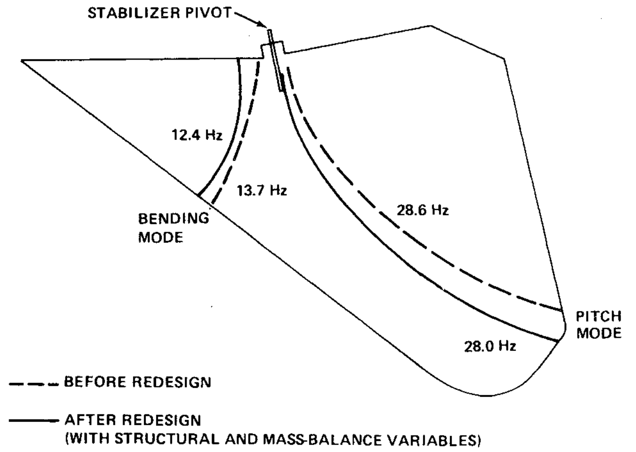

Fig. 5 All-movable stabilizer vibration mode node lines before and after redesign.

design so as to achieve a significant flutter-speed increase. For the composite wing, the program was used to resize the individual ply thicknesses of a strength-based design for increased flutter speed, considering the gages of the initial fully stressed structure as minimum allowable gages, i.e., noninteractive resizing. It should be noted that in each example all flutter speeds are computed at constant Mach number and altitude, and hence do not represent matched-point conditions.

All-Movable Stabilizer

The structures model of all-movable stabilizer, illustrated in Fig. 2, was modeled with 890 finite elements. (Details of the actuator restraint and actuator horn are omitted from the figure for clarity.) The stabilizer construction consisted of titanium covers, modeled as membrane elements, and a full-depth aluminum honeycomb core, modeled as spanwise and chordwise shear panels having equivalent stiffness properties. Since the shear panels logically could not be resized for strength or flutter requirements, they were not included in the redesign process, thereby leaving 324 structural design variables.

A supersonic flutter-critical flight condition (Mach 1.6, 30,000 ft) was selected for flutter redesign, with the design goal being to increase the flutter speed of the strength-based design by 25%. Two separate studies were performed, one using both structural and mass-balance design variables and the other using only structural design variables. In the former case, 3 lb of initial mass balance was added arbitrarily at each of the three selected mass-balance locations illustrated in Fig. 2. Reasonably large values of mass were selected to avoid the possibility, observed in previous studies, of mass balance being ineffective for low initial values even though it becomes effective for larger values.

The results of the combined strength/flutter redesign study including mass-balance variables, showed that the mass balance at point *B* was increased beyond its initial value, whereas the masses at points *A* and *C* were eliminated progressively. After the third flutter redesign cycle, it was obvious that the mass balance at points *A* and *C* would vanish in the final design. Accordingly, this mass balance was eliminated at that stage of redesign, thereby accelerating convergence to the optimum design.

Figure 3 shows that a near-optimum design with combined structural and mass-balance variables was achieved after only five redesign cycles. The net weight increase to achieve a 25% increase in flutter speed was only 10.70 lb, or 2.6% of the total stabilizer weight. Three additional redesign cycles beyond this point eliminated an additional 0.45 lb – a relatively insignificant amount. The distribution of mass balance and structural weight in the final design, presented in Fig. 4, indicates that structural stiffening occurred in the vicinity of the root rib, to increase the stiffness of both the apex region and the stabilizer actuator-horn attachment points. As indicated in Fig. 5, redesign caused: 1) a significant forward movement of the vibration-mode node lines in both the bending and pitch modes, the two modes active in the flutter mechanism, and 2) increased separation of bending- and torsion-mode frequencies. These effects were due primarily to the addition of mass balance. In addition, the apex stiffening apparently caused the pitch mode frequency to drop less rapidly with increasing airspeed, thereby effecting a further flutter-speed increase. Table 2 includes a summary of design data for this study, wherein it is noted that resizing for strength (a single cycle of FSD in each combined resizing cycle) resulted in a very small reduction in the weight of the strength-critical structure.

The second study, which used only structural design variables, indicated that the flutter effectiveness of the structural elements in the tip region was governed by their mass-balance contribution rather than their effect on structural stiffness. Consequently, FASTOP resized these elements to achieve a mass-balance effect similar to that noted in the previous study. Since the initial values for the mass of these minimum-gage structural elements were very small, it became apparent that convergence to the previously obtained optimum combination of "mass balance" and structural stiffness would be slow (see upper right portion of Fig. 3). Redesign was, therefore, terminated before this convergence was achieved. This case illustrates a point noted in previous studies using FASTOP, namely, that for flutter cases that are susceptible to mass balance, convergence is much enhanced if the user initiates redesign with arbitrary selections of mass balance in potentially effective regions of the structure.

To provide an additional base of reference for the two studies just described, a final analysis was performed (see Fig. 3) using mass balance alone at the most effective location indicated by FASTOP, i.e., the tip leading edge. It was determined that 14.7 lb of mass balance would be required to achieve the 25% flutter-speed increase, compared with 10.25 lb of total mass balance and structural weight for the combined redesign case.

Wing with Store

The structures model of the wing with pylon-mounted store, illustrated in Fig. 6, was modeled with approximately 600 finite elements. The idealization of the multiple-spar aluminum wing box used quadrilateral membrane elements to represent the wing covers and quadrilateral shear panels for rib and spar webs. Upper- and lower-cover nodes were connected with bar or "post" elements. The semiwing was modeled from the tip to the airplane centerline, and symmetric boundary conditions were specified for all structural nodes in the plane of symmetry. The wing-store pylon was

Table 2 Final design data for three demonstration cases

	ALL-MOVABLE STABILIZER	WING-WITH-STORE	COMPOSITE WING
WEIGHT OF FINITE-ELEMENT MODEL FOR INITIAL FULLY STRESSED DESIGN*	220.2	1340	40.9
TOTAL WEIGHT OF FSD STRUCTURE INCLUDING NON-OPTIMUM FACTORS	414.5	1921	187.3
TOTAL WEIGHT INCREMENT OF FLUTTER-CRITICAL ELEMENTS FROM FSD (FINAL DESIGN)	STRUCTURE = 3.71 MASS BALANCE = 6.78	PYLON STRUCTURE = 6.55 WING STRUCTURE = 3.80	WING STRUCTURE = 7.5
TOTAL WEIGHT CHANGE OF STRENGTH-CRITICAL ELEMENTS FROM FSD	-0.24	-1.93	DOES NOT APPLY
TOTAL WEIGHT INCREMENT FROM FSD	10.25	8.42	7.5
FLUTTER SPEED CHANGE (KEAS) (PERCENTAGE INCREASE)	665 → 831 (25)	270 → 660 (144)	752 → 978 (30)

*All weights in pounds.

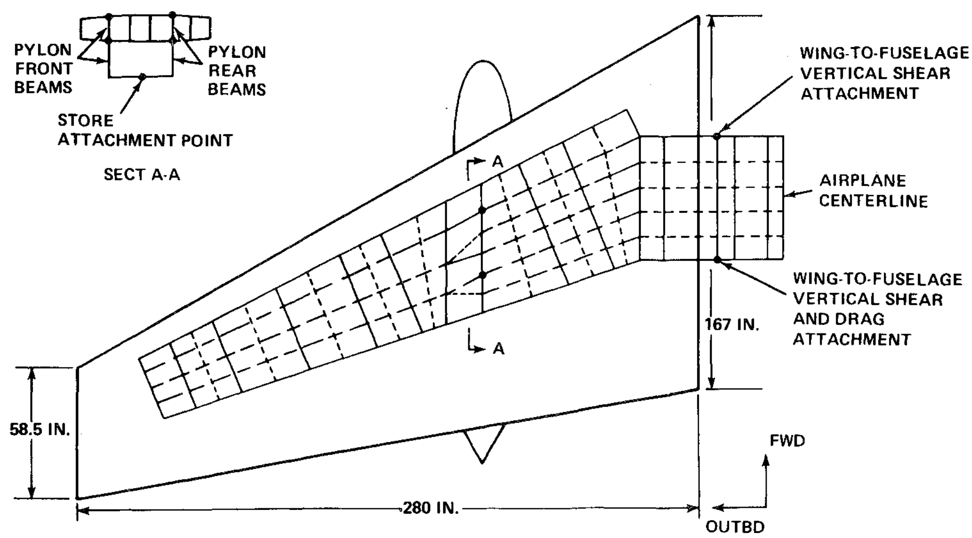

Fig. 6 Wing-with-store structures model.

modeled with beam elements attached to upper- and lower-cover wing node points. A symmetric free-free vibration model was used for flutter analysis, the fuselage being dynamically modeled with beam elements which simulated the stiffness properties of the fuselage in vertical bending. The weight of the initial fully stressed wing structure was 1340 lb, based on the finite-element idealization, and 1921 lb, including nonoptimum factors and overhanging structure. The pylon-mounted store weighed 4500 lb, with a pitch inertia of 8×10^6 lb-in.^{2} about its center of gravity. This high store inertia created a critical flutter mechanism involving the first wing bending mode and the store-pitch/wing-torsion mode. A Mach 0.8, sea level, flutter-critical flight condition was designated for the redesign study, the objective being to achieve a flutter-speed target of 660 knots for the fully stressed design. Wing posts and fueslage beam elements were excluded from strength/flutter redesign, resulting in a total of 453 active structural design variables for this study.

In the initial design, the front beam of the pylon support structure had the highest flutter-velocity derivative (236 knots/lb), and the beam element which continued the front beam connection from the lower to the upper wing cover had the second highest derivative (97 knots/lb). For the first resizing steps, the rear beam of the pylon, which had an initial flutter-velocity derivative of -41.5 knots/lb, was not resized downward because of minimum-gage constraints. As redesign proceeded, however, the front and rear beams of the pylon became equally flutter-effective. This change was accompanied by a noticeable change in the flutter mode shape, which initially involved components of the first wing bending mode and the store-pitch/wing-torsion mode, but later exhibited increasingly large components of the second wing bending mode as the store-pitch frequency was increased. It may be noted, in Fig. 7, that a net reduction in structural weight was achieved in the first combined strength/flutter redesign cycle. This was because the initial fully stressed design was not converged fully, and the subsequent strength redesign further reduced the weight of the structure in regions that were ineffective for flutter. The net weight reduction of 0.95 lb in the first redesign cycle comprised an addition of 0.34 lb for flutter-speed improvement and a reduction of 1.29 lb in the strength-critical regions of the structure.

Subsequent redesign for increased flutter speed appeared to be directed toward increasing the frequency of the store-pitch mode, thereby separating the first-bending and store-pitch frequencies as illustrated in Fig. 8. No structural elements were resized outboard of the wing store station. Resizing of wing structural elements inboard of the store station, illustrated in Fig. 9, involved spar webs and the rib webs between the wing-to-pylon connection points. The resizing in the vicinity of the front wing-to-fuselage connection point accounted for a relatively small proportion of the overall weight increase. One of the more interesting results of this study was that resizing to increase overall store-pitch stiffness

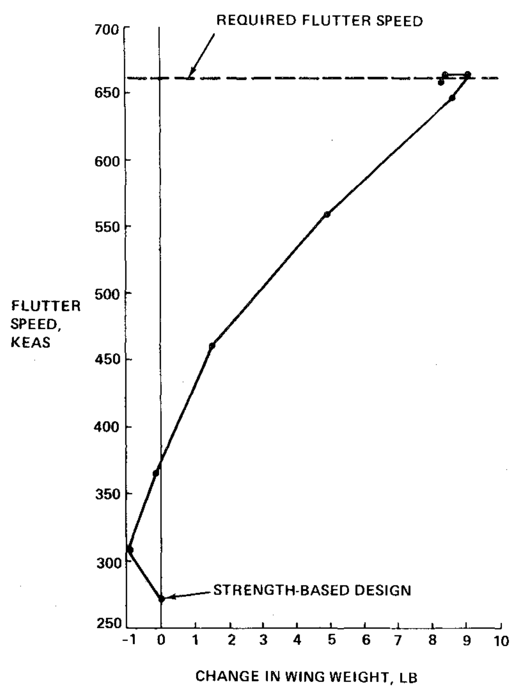

Fig. 7 Results of wing-with-store redesign study.

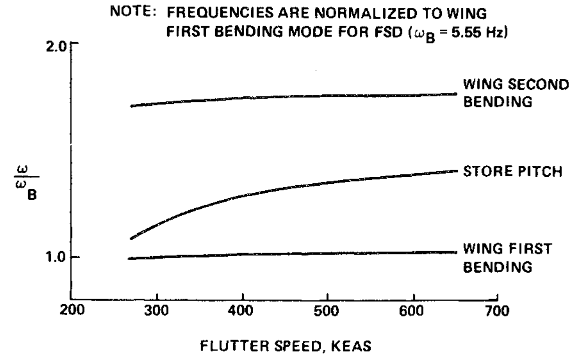

Fig. 8 Wing-with-store: variation of modal frequencies during redesign.

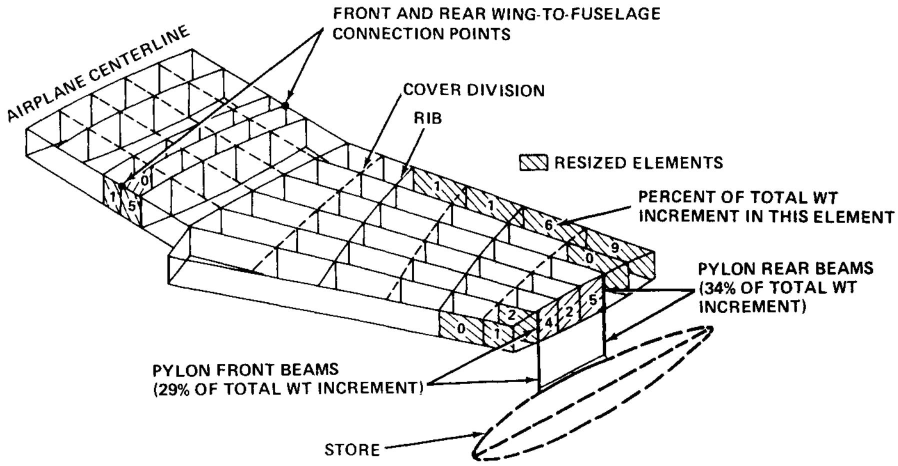

Fig. 9 Design changes in wing-with-store.

was achieved more efficiently by increasing the gages of the wing spar webs than by resizing the covers.

As indicated in Fig. 7, convergence to the final design point was achieved in eight combined strength/flutter redesign cycles, although a design very close to optimum was obtained in only six cycles. The flutter-speed target was achieved for a weight increase of 8.42 lb. A summary of pertinent design data is given in Table 2. From a flutter standpoint, the most significant effects of redesign on the wing vibration mode characteristics appeared to be the previously mentioned increase in frequency of the store-pitch mode and the substantial forward shift of the node line in that mode, as shown in Fig. 10.

Although the net reduction in weight of the strength-critical structure was relatively insignificant, a redistribution of cover material was noted in the inboard wing sections. Consequently, strength resizing was performed in most of the redesign cycles. However, in two cycles the structural optimization module of FASTOP was used simply to recompute a revised flexibility matrix for vibration-mode calculations.

Composite Wing

The aerodynamic planform and primary structural arrangement for the composite wing example are shown in Fig. 11, along with the four fiber directions of the laminated cover skins. The primary structure was a two-cell, symmetric box beam that was built-in at the root. Its graphite/epoxy cover skins were modeled with stacked orthotropic membrane elements representing material in each of the four fiber directions. The aluminum substructure was modeled with shear-panel elements and posts.

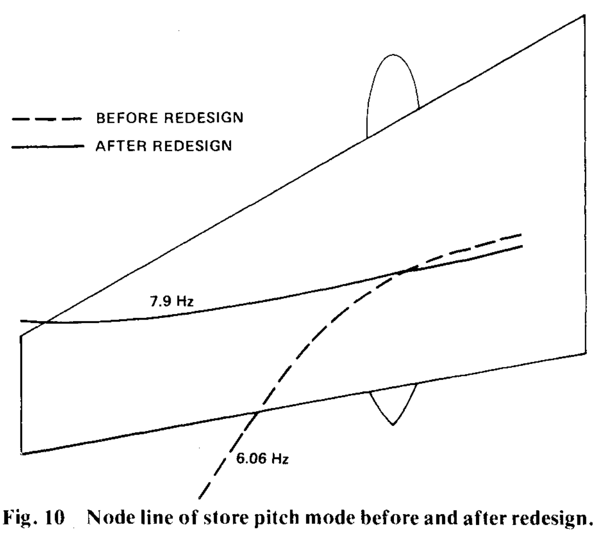

Fig. 10 Node line of store pitch mode before and after redesign.

Since the strength resizing module in the present version of FASTOP cannot deal with composite materials of general fiber orientation, the initial design-load computation and strength sizing for the wing were achieved by using selected programs from Grumman's Rapid Aerospace Vehicle Evaluation System (RAVES).[5] FASTOP then was used to compute a dynamics model flexibility matrix and normal modes of vibration. A flutter analysis, using FASTOP's subsonic doublet-lattice program with $M = 0.8$ at sea level and the six lowest-frequency normal modes as generalized coordinates, indicated that the flutter speed of the strength-based design was 752 knots. The critical flutter mechanism resulted from coupling between the fundamental bending and torsion modes.

The program then was used to achieve a 30% increase in flutter speed by considering the thickness of each membrane element (representing a particular fiber orientation) as a discrete design variable. Only the cover-skin elements were allowed to be resized and no element thickness could be reduced below its original strength-based or minimum-gage value. The model contained 256 design variables. Figure 12 summarizes the results of the resizing steps, from the initial

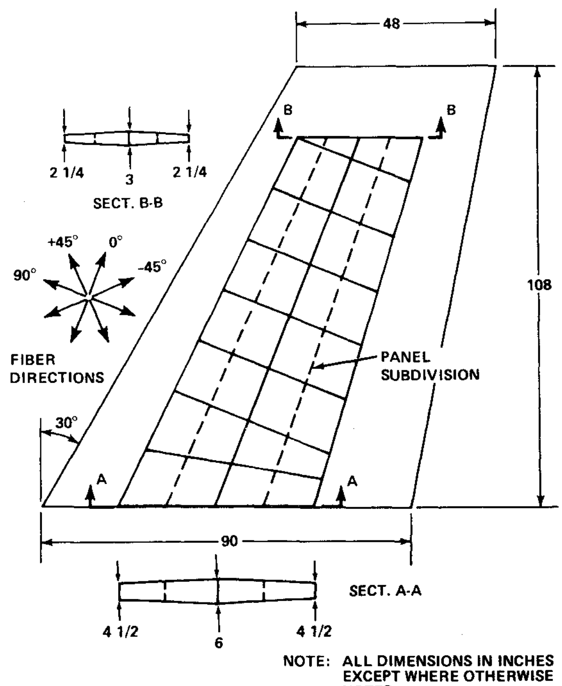

Fig. 11 Aerodynamic planform and primary structural arrangement of composite wing.

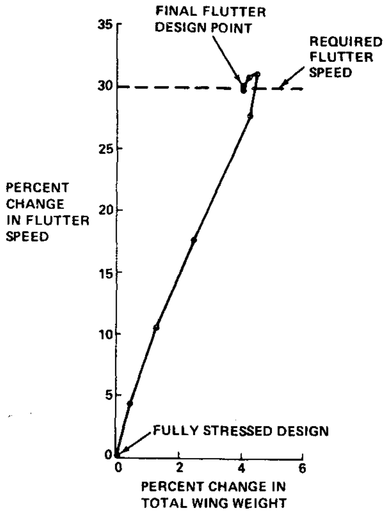

Fig. 12 Results of composite-wing redesign study.

PERCENTAGES SHOWN ARE FOR BOTH UPPER AND LOWER COVERS

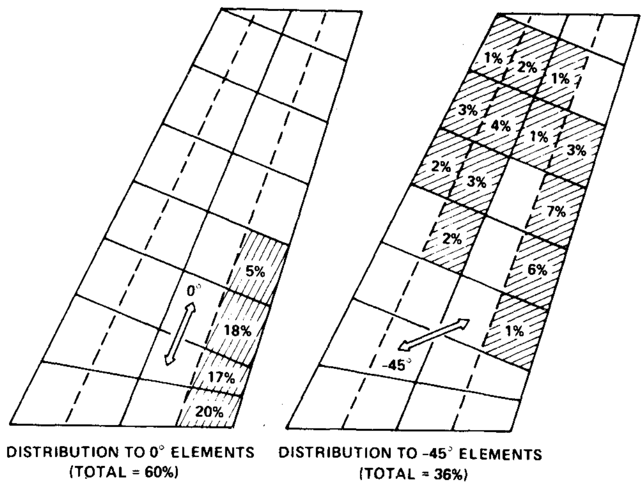

Fig. 13 Distribution of total weight increment added to composite cover skins for increased flutter speed.

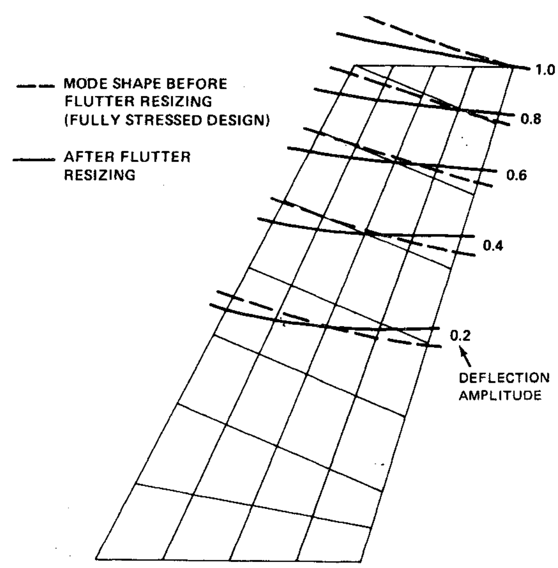

Fig. 14 Comparison of fundamental wing bending mode of composite wing before and after resizing for flutter.

strength-based design to the minimum-weight flutter design. It can be seen that convergence was achieved in eight resizing steps, with a weight increment of 4% of the original total wing weight (or about 7.5 lb). Figure 13 shows how most of this weight increment was distributed among the elements that were resized to meet the flutter-speed requirement. Of this total increment, 60% went into the 0° elements in the trailing-edge root region and 36% went into the −45° elements, largely in the outboard half of the wing.

An examination of the wing vibration mode shapes, before and after redesign, indicated that the torsion mode was not changed significantly, but there was a notable change in the primary bending mode, as illustrated in Fig. 14. In the strength-based design, bending occurred along the swept axis of the wing box beam, whereas, after redesign, bending was more along an axis perpendicular to the streamwise direction. The net effect of this change was to reduce the tip angle of attack in the primary bending mode by about 50%. This change resulted in less aerodynamic stiffening in this mode, making its frequency less sensitive to airspeed, and consequently, delaying the onset of flutter. Results of this final study are summarized in Table 2.

Conclusions

The three studies discussed herein demonstrate that FASTOP provides an efficient approach to the sizing of metallic structures for combined strength and flutter requirements and, also, composite structures for flutter requirements. In all of the cases studied, the program system was able to achieve near-minimum-weight designs with only a small number of resizing cycles. It is highly unlikely that such efficient designs could have been obtained by conventional, nonautomated, flutter-prevention procedures. The demonstrated ability to handle a diversity of structural configurations provides assurance that this automated procedure is a useful tool that can be applied in the real design environment.

Acknowledgment

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